Warm-Up

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CST/CAHSEE: Algebra I 15.0

One pipe can fill a tank in 20 minutes, while another takes 30 minutes to fill the same tank. How long would it take the two pipes together to fill the tank?

A 50 min

B 25 min

C 15 min

D 12 min

Don't Solve Yet!

Which 2 answers can you eliminate immediately?

Why?

Review: Grade 7 MG 1.3

A utility company estimates that a power line repair job will take a total of 24 person-hours. If 3 workers are assigned to the job, how long will it take them to complete the job according to this estimate?

A 8 hours

B 12 hours

C 27 hours

D 72 hours

Which 2 answers can you eliminate immediately?

Why?

x

Current: Algebra I 5.0

Solve using at least 2 different methods:

$$\left(\frac{1}{3}+\frac{1}{6}\right)x=1$$

Other: Grade 6 AF 2.2

A water tank will hold 50 gallons. What flow rate, in gallons per second, is required to fill the tank in 20 seconds?

Solving Algebra I Work Problems Beginning With a Bar Model

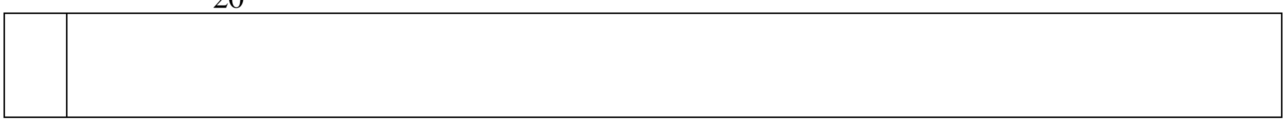
In Algebra I the work problems involve either people or machines doing a job. The jobs are either the same or equivalent and the focus will be on how much of the job can be completed in one unit of time (seconds, minutes, hours, etc.).

Example 1 (the CST question that was part of the warm-up)
One pipe can fill a tank in 20 minutes, while another takes 30 minutes to fill the same tank. How long would it take the two pipes together to fill the same tank?

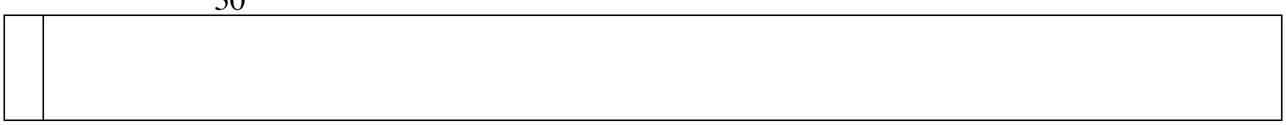
Draw four bars each representing the entire tank. The first two bars represent the amount each pipe could fill the tank in one unit of time (a minute). The third bar represents the amount the two pipes could fill the tank together in one unit of time (a minute). The fourth bar represents the actual time it will take the pipes to fill the tank together.

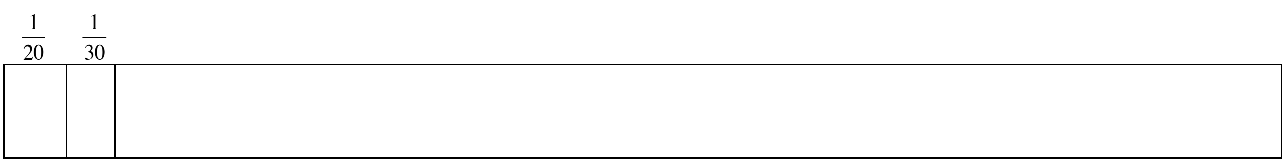
Based on the discussion of the enhanced question in the warm-up students should understand that together the pipes have to fill the tank faster than the faster pipe could by itself. So together, they have to be able to fill the tank in less than 20 minutes.

Pipe 1 can fill $\frac{1}{20}$ of the tank in 1 minute.

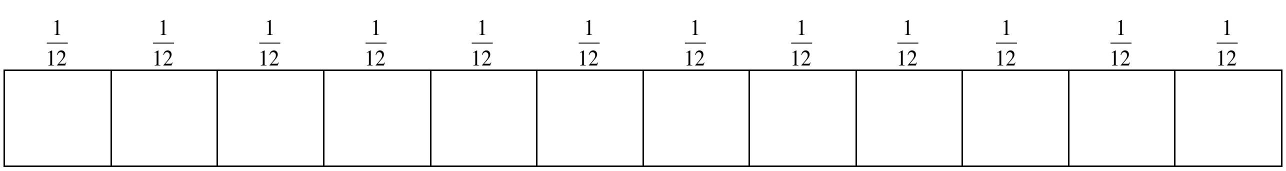


Pipe 2 can fill $\frac{1}{30}$ of the tank in 1 minute.





Together they can fill $\frac{1}{20}$ plus $\frac{1}{30}$ or $\frac{1}{12}$ of the tank in 1 minute.



Each $\frac{1}{12}$ is equivalent to 1 minute.

Therefore it will take 12 minutes for the pipes to fill the tank together.

This will help the students to understand conceptually what is taking place in these types of problems even when the problems do not end up equaling a whole number. Also, be aware that there are many different ways to show bar models.

When we set up the equation, we need to define our variable.

Let x = the amount of time the two pipes take to fill the tank together

Next if we think of the fractional amount that one pipe can fill the tank in one minute, $\frac{1}{20}$, plus the fractional amount that the other pipe can fill the tank in one minute, $\frac{1}{30}$, and multiply the sum by the amount of time it takes the two pipes to fill the tank together then it would equal the 1 full tank.

$$\left(\frac{1}{20}+\frac{1}{30}\right)x=1$$

We can then solve the equation multiple ways.

$$\left(\frac{1}{20}+\frac{1}{30}\right)x=1$$

$$\begin{aligned}\left(\frac{1}{20}+\frac{1}{30}\right)x&=1\\ \left[\left(\frac{1}{20}\bullet\frac{3}{3}\right)+\left(\frac{1}{30}\bullet\frac{2}{2}\right)\right]x&=1\\ \left(\frac{3}{60}+\frac{2}{60}\right)x&=1\\ \left(\frac{3+2}{60}\right)x&=1\\ \frac{5}{60}x&=1\\ \frac{5}{5\bullet 12}x&=1\\ \frac{1}{12}x&=1\\ \left(\frac{12}{1}\right)\frac{1}{12}x&=1\left(\frac{12}{1}\right)\\ \boxed{x=12}\end{aligned}$$

$$\begin{aligned}\left(\frac{1}{20}+\frac{1}{30}\right)x&=1\\ \frac{1}{20}x+\frac{1}{30}x&=1\\ \frac{3}{60}x+\frac{2}{60}x&=1\\ \frac{5}{60}x&=1\\ \frac{5}{5\bullet 12}x&=1\\ \frac{1}{12}x&=1\\ 12\bullet\frac{1}{12}x&=1\bullet 12\\ \boxed{x=12}\end{aligned}$$

$$\begin{aligned}\left(\frac{1}{20}+\frac{1}{30}\right)x&=1\\ 60\left[\left(\frac{1}{20}+\frac{1}{30}\right)x\right]&=1\bullet 60\\ (3+2)x&=60\\ 5x&=60\\ \frac{5x}{5}&=\frac{60}{5}\\ \boxed{x=12}\end{aligned}$$

$\therefore$ It would take the two pipes 12 minutes to fill the tank together.

You Try 1

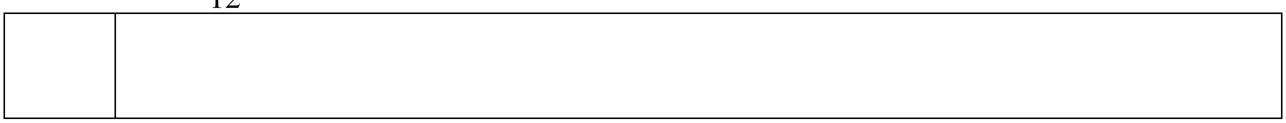
Vanessa can paint the living room in 10 hours and Cris can paint the room in 12 hours. How long would it take Vanessa and Cris to paint the room together?

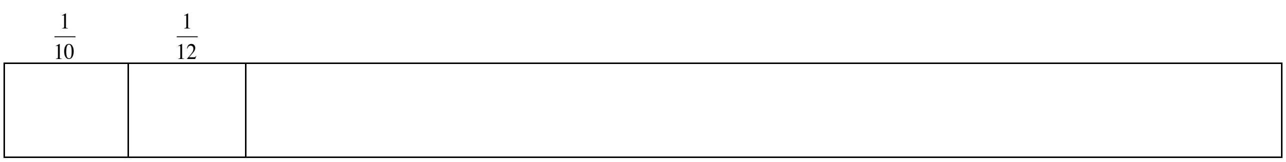
Draw the four bars each representing the entire room. The first two bars represent the amount each person could paint in one unit of time (an hour). The third bar represents the amount they could paint together in one unit of time (an hour). The fourth bar represents the actual time it will take them to paint the room together.

Continue to ask the students what we know about how long they can paint the room together (it has to be less than 10 hours).

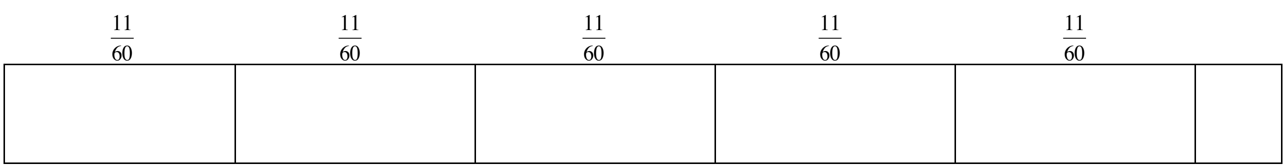
Vanessa can paint $\frac{1}{10}$ of the room in 1 hour.

Cris can paint $\frac{1}{12}$ of the room in 1 hour.





Together they can paint $\frac{1}{10}$ plus $\frac{1}{12}$ or $\frac{11}{60}$ of the room in 1 hour.



Each $\frac{11}{60}$ represents 1 hour. Therefore, these five $\frac{11}{60}$ are equivalent to 5 hours.

From this we know it has to be more than 5 but less than 6 hours. The remaining amount is $\frac{60}{60}-\frac{55}{60}$ or $\frac{5}{60}$. So this remaining amount of time is how many $\frac{11}{60}$ go into $\frac{5}{60}$.

$$\frac{5}{60}\div\frac{11}{60} = \frac{5\div 11}{60\div 60} = \frac{\frac{5}{11}}{1} = \boxed{\frac{5}{11}}$$

$$\frac{1}{12}\div\frac{11}{60} = \frac{1}{12}\bullet\frac{60}{11} = \frac{5\bullet \cancel{12}}{\cancel{12}\bullet 11} = \boxed{\frac{5}{11}}$$

$\therefore$ Together it will take $5\frac{5}{11}$ hours.

When we set up the equation, we need to define our variable.

Let x = the amount of time the two can paint the room together

$$\left(\frac{1}{10}+\frac{1}{12}\right)x=1$$

$$\left(\frac{1}{10}+\frac{1}{12}\right)x=1$$

$$60\left[\left(\frac{1}{10}+\frac{1}{12}\right)x\right]=[1]60$$

$$(6+5)x=60$$

$$11x=60$$

$$\frac{11x}{11}=\frac{60}{11}$$

$$\boxed{x=5\frac{5}{11}}$$

$$\left(\frac{1}{10}+\frac{1}{12}\right)x=1$$

$$\frac{11}{60}x=1$$

$$\left(\frac{60}{11}\right)\frac{11}{60}x=1\left(\frac{60}{11}\right)$$

$$x=\frac{60}{11}$$

$$\boxed{x=5\frac{5}{11}}$$

$\therefore$ It would take them $5\frac{5}{11}$ hours to paint the room together.

Example 2

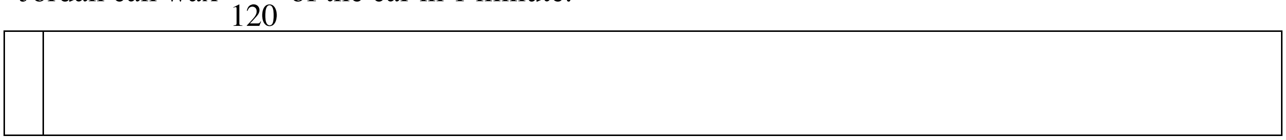
Jordan can wax his car in 2 hours. When he works together with Juanita, they can wax the car in 45 minutes. How long would it take Juanita working by herself to wax the same car?

Notice that this problem is giving different information than the first example. This problem gives the time it takes one person to do the job and the time it takes both working together to do the job. It is asking for the time it takes the other person to do the job by themselves.

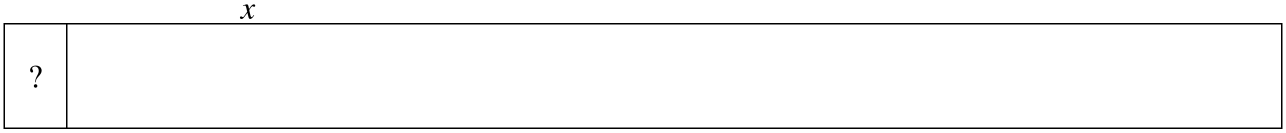
Also, we need to convert one of the times given so that we are working with the same units.

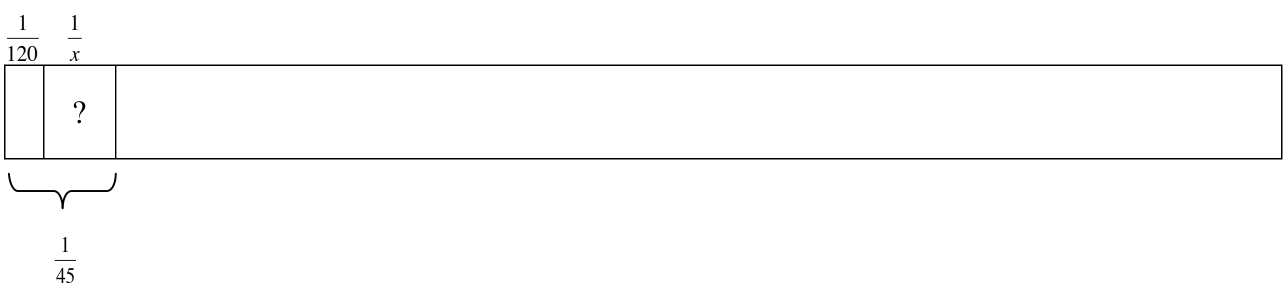
First, set up the four bars each representing the entire car that is being waxed. Continue to ask the students what we know about how long it will take Juanita (it has to be longer than 45 minutes).

Jordan can wax $\frac{1}{120}$ of the car in 1 minute.



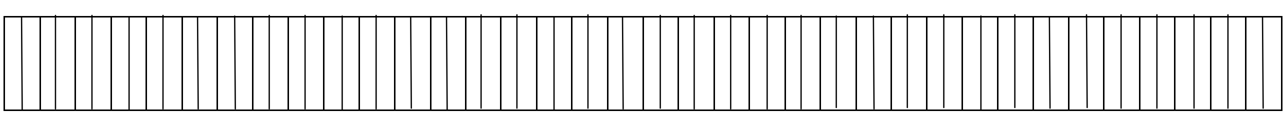
Juanita can wax $\frac{1}{x}$ of the car in 1 minute.





Together they can wax $\frac{1}{45}$ of the car in 1 minute.

Juanita can wax $\frac{1}{45} - \frac{1}{120}$ or $\frac{1}{72}$ of the car in 1 minute.



Each bar above represents $\frac{1}{72}$ of the entire car being waxed in 1 minute.

Therefore, Juanita can wax the car by herself in 72 minutes.

When we set up the equation, we need to define our variable.

Let x = the amount of time it takes Juanita to wax the car

Then set up the equation the same way we set up the equation in the previous example and you try. Add the fractional part of times for both people and multiply that sum by the total amount of time it takes them together and set it equal to the one job of waxing the car.

$$\left(\frac{1}{120}+\frac{1}{x}\right)45=1$$

Again, the equation can be solved many ways.

$$\begin{aligned}
\left(\frac{1}{120}+\frac{1}{x}\right)45&=1\\
\frac{45}{120}+\frac{45}{x}&=1\\
\frac{3}{8}+\frac{45}{x}&=1\\
(8x)\frac{3}{8}+\frac{45}{x}&=1(8x)\\
3x+360&=8x\\
3x-3x+360&=8x-3x\\
360&=5x\\
\frac{360}{5}&=\frac{5x}{5}\\
\boxed{72=x}
\end{aligned}$$

$$\begin{aligned}
\left(\frac{1}{120}+\frac{1}{x}\right)45&=1\\
\frac{\left(\frac{1}{120}+\frac{1}{x}\right)45}{45}&=\frac{1}{45}\\
360x\left(\frac{1}{120}+\frac{1}{x}\right)&=\left(\frac{1}{45}\right)360x\\
3x+360&=8x\\
\cancel{3x}+360&=\cancel{3x}+5x\\
72+72+72+72+72&=x+x+x+x+x\\
\boxed{72=x}
\end{aligned}$$

$\therefore$ It would take Juanita 72 minutes to wax the car herself.

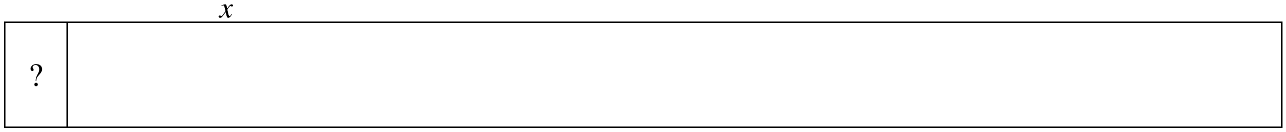
You Try 2

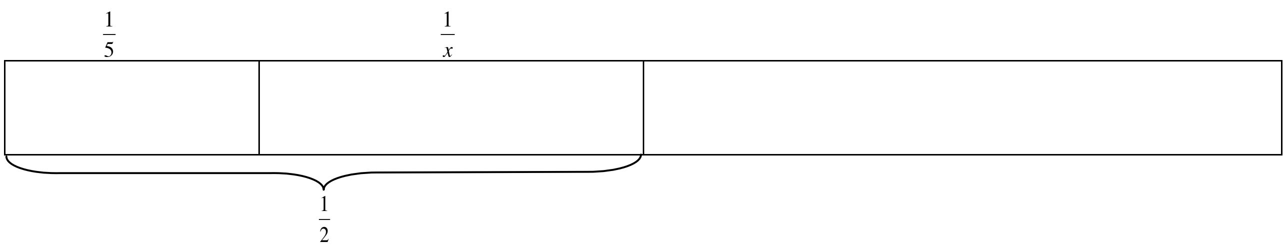
If Leslie can type a paper in 5 hours and together Leslie and Chan can type the paper in 2 hours, how long will it take Chan to type the same paper by himself?

Draw the bar models to help with the conceptual understanding. Continue to ask the students what we know about how long it will take Chan (it has to be more than 2 hours).

Leslie can type $\frac{1}{5}$ of the paper in 1 hour.

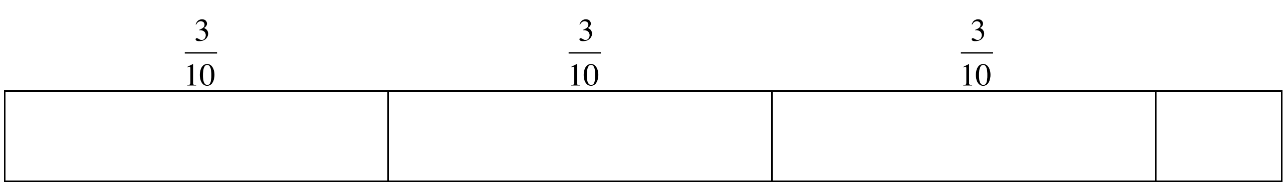
Chan can type $\frac{1}{x}$ of the paper in 1 hour.





Together they can type $\frac{1}{2}$ of the paper in 1 hour.

Chan can type $\frac{1}{2}-\frac{1}{5}$ or $\frac{3}{10}$ of the paper in 1 hour.



Each $\frac{3}{10}$ represents 1 hour. Therefore, these three $\frac{3}{10}$ are equivalent to 3 hours.

From this we know it has to be more than 3 but less than 4 hours. The remaining amount is $\frac{10}{10}-\frac{9}{10}$ or $\frac{1}{10}$. So this remaining amount of time is how many $\frac{3}{10}$ go into $\frac{1}{10}$.

$$\frac{1}{10}\div\frac{3}{10}$$
$$=\frac{1\div 3}{10\div 10}$$
$$=\frac{\frac{1}{3}}{1}$$
$$=\boxed{\frac{1}{3}}$$

$$\frac{1}{10}\div\frac{3}{10}$$
$$=\frac{1}{10}\bullet\frac{10}{3}$$
$$=\frac{\cancel{10}}{\cancel{10}\bullet 3}$$
$$=\boxed{\frac{1}{3}}$$

$\therefore$ Chan can type the paper in $3\frac{1}{3}$ hours.

When we set up the equation, we need to define our variable.

Let x = the amount of time it takes Chan to type the paper by himself

$$\left(\frac{1}{5}+\frac{1}{x}\right)2=1$$
$$\frac{\left(\frac{1}{5}+\frac{1}{x}\right)2}{2}=\frac{1}{2}$$
$$\frac{1}{5}+\frac{1}{x}=\frac{1}{2}$$
$$(10x)\frac{1}{5}+\frac{1}{x}=\frac{1}{2}(10x)$$
$$2x+10=5x$$
$$2x-2x+10=5x-2x$$
$$10=3x$$
$$\frac{10}{3}=\frac{3x}{3}$$
$$\boxed{3\frac{1}{3}=x}$$

$$\left(\frac{1}{5}+\frac{1}{x}\right)2=1$$
$$\frac{2}{5}+\frac{2}{x}=1$$
$$5x\left(\frac{2}{5}+\frac{2}{x}\right)=(1)5x$$
$$2x+10=5x$$
$$2x+10=2x+3x$$
$$10=3x$$
$$\frac{10}{3}=\frac{3x}{3}$$
$$\boxed{3\frac{1}{3}=x}$$

$\therefore$ It would take Chan $3\frac{1}{3}$ hours to type the paper himself.

Example 3

One carpet layer can install 50 ft^2 in 5 hours and another carpet layer can install 50 ft^2 in 3 hours. How many ft^2 could they install working together in 9 hours?

First Carpet Installer

Hour 1	Hour 2	Hour 3	Hour 4	Hour 5
10 ft^2	10 ft^2	10 ft^2	10 ft^2	10 ft^2

Second Carpet Installer

Hour 1	Hour 2	Hour 3
$\frac{50}{3}\text{ ft}^2$	$\frac{50}{3}\text{ ft}^2$	$\frac{50}{3}\text{ ft}^2$

Working Together

Hour 1	Hour 2	Hour 3	Hour 4	Hour 5	Hour 6	Hour 7	Hour 8	Hour 9
$10\text{ ft}^2+\frac{50}{3}\text{ ft}^2$	$10\text{ ft}^2+\frac{50}{3}\text{ ft}^2$	$10\text{ ft}^2+\frac{50}{3}\text{ ft}^2$	$10\text{ ft}^2+\frac{50}{3}\text{ ft}^2$	$10\text{ ft}^2+\frac{50}{3}\text{ ft}^2$	$10\text{ ft}^2+\frac{50}{3}\text{ ft}^2$	$10\text{ ft}^2+\frac{50}{3}\text{ ft}^2$	$10\text{ ft}^2+\frac{50}{3}\text{ ft}^2$	$10\text{ ft}^2+\frac{50}{3}\text{ ft}^2$

Using the bar model we know that each hour they can install $10\text{ ft}^2+\frac{50}{3}\text{ ft}^2$ and if we multiply that by the 9 hours they work together, the result would be the total square feet of carpet they could install.

Let x = the square feet of carpet they could install together in 9 hours

$$(10\text{ ft}^2+\frac{50}{3}\text{ ft}^2)9=x$$

$$90\text{ ft}^2+150\text{ ft}^2=x$$

$$\boxed{240\text{ ft}^2=x}$$

$\therefore$ They could install 240 ft^2 of carpet in 9 hours.

Did you notice that this equation was set up the same as all of the other work problems.

The sum of the two workers' unit rates multiplied by the time they work together is equal to the entire job.

The difference with this example is that the entire job is not 1 but rather the amount of work that they could complete in a given time.

You Try 3

Joan can lay 10 ft^2 of tile in 2 hours while Bobby can lay 10 ft^2 of tile in 3 hours. How much tile could they lay together in 6 hours?

Joan

5 ft^2	5 ft^2
Hour 1	Hour 2

Robert

$\frac{10}{3} \text{ ft}^2$	$\frac{10}{3} \text{ ft}^2$	$\frac{10}{3} \text{ ft}^2$
Hour 1	Hour 2	Hour 3

Working Together

$5 \text{ ft}^2 + \frac{10}{3} \text{ ft}^2$	$5 \text{ ft}^2 + \frac{10}{3} \text{ ft}^2$	$5 \text{ ft}^2 + \frac{10}{3} \text{ ft}^2$	$5 \text{ ft}^2 + \frac{10}{3} \text{ ft}^2$	$5 \text{ ft}^2 + \frac{10}{3} \text{ ft}^2$	$5 \text{ ft}^2 + \frac{10}{3} \text{ ft}^2$
Hour 1	Hour 2	Hour 3	Hour 4	Hour 5	Hour 6

Using the bar model we know that each hour they can install $5 \text{ ft}^2 + \frac{10}{3} \text{ ft}^2$ and if we multiply that by the 6 hours they work together, the result would be the total square feet of tile they could install.

Let x = the square feet of tile they could install together in 6 hours

$$\left(5 \text{ ft}^2 + \frac{10}{3} \text{ ft}^2\right)6 = x$$

$$30 \text{ ft}^2 + 20 \text{ ft}^2 = x$$

$$\boxed{50 \text{ ft}^2 = x}$$

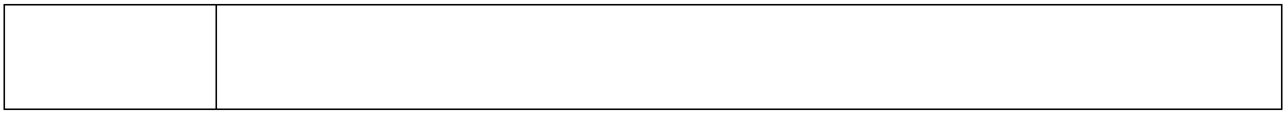
$\therefore$ They could lay 50 ft^2 of tile in 6 hours.

Once again, the sum of the two workers' unit rates multiplied by the time they work together is equal to the entire job.

Example 4

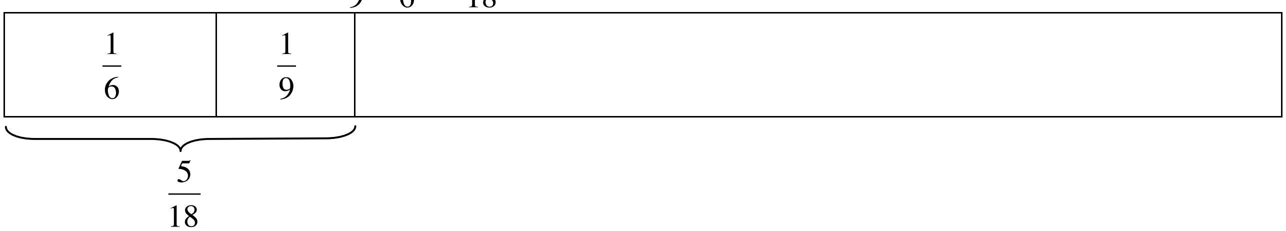
Jalen can set up all the seating in the cafeteria in 9 minutes. Lizette can set up all the seating in 6 minutes. If Lizette begins to set up the seats in the cafeteria by herself for 1 minute and is then helped by Jalen, how long will it take them together to finish setting up the seating?

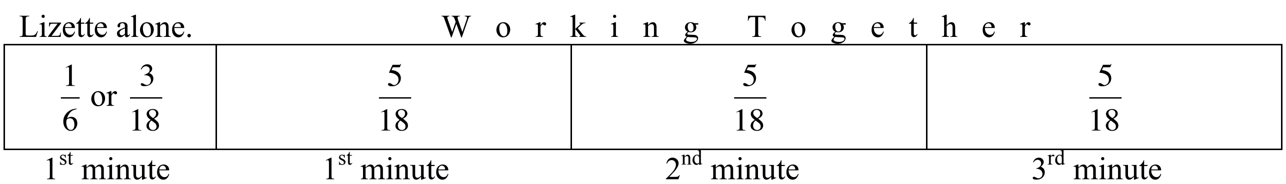
Lizette can set up $\frac{1}{6}$ of the seats in one minute.



Jalen can set up $\frac{1}{9}$ of the seats in one minute.

Together they can set up $\frac{1}{9}+\frac{1}{6}$ or $\frac{5}{18}$ of the seats in one minute.





Therefore, it will take them 3 minutes working together to finish setting up the seats in the cafeteria.

When we set up the equation, we need to define our variable.

Let x = the amount of time it takes Lizette and Jalen to finish setting up the seats in the cafeteria

$$\left(\frac{1}{6}\right)1+\left(\frac{1}{9}+\frac{1}{6}\right)x=1$$
$$\frac{1}{6}-\frac{1}{6}+\left(\frac{5}{18}\right)x=1-\frac{1}{6}$$
$$\frac{5}{18}x=\frac{5}{6}$$
$$\left(\frac{18}{5}\right)\frac{5}{18}x=\frac{5}{6}\left(\frac{18}{5}\right)$$
$$\frac{18\bullet 5}{5\bullet 18}x=\frac{5\bullet 6\bullet 3}{6\bullet 5}$$
$$\boxed{x=3}$$

$$\left(\frac{1}{6}\right)1+\left(\frac{1}{9}+\frac{1}{6}\right)x=1$$
$$\left(\frac{1}{6}\right)1+\left(\frac{1}{9}\right)x+\left(\frac{1}{6}\right)x=1$$
$$\left(\frac{1}{6}\right)x+\left(\frac{1}{6}\right)1+\left(\frac{1}{9}\right)x=1$$
$$\frac{1}{6}(x+1)+\left(\frac{1}{9}\right)x=1$$
$$54\left[\frac{1}{6}x+\frac{1}{6}+\frac{1}{9}x\right]=[1]54$$
$$9x+9+6x=54$$
$$15x+9-9=54-9$$
$$15x=45$$
$$\frac{15x}{15}=\frac{15\bullet 3}{15}$$
$$\boxed{x=3}$$

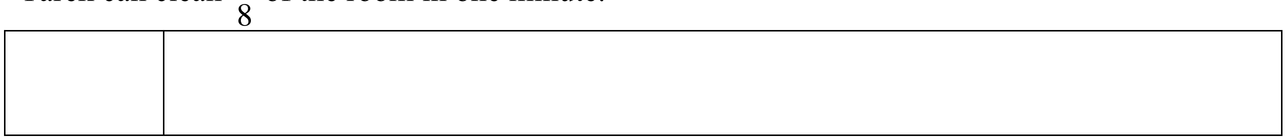
∴ It will take Lizette and Jalen 3 minutes to finish setting up the seats in the cafeteria.

You Try 4

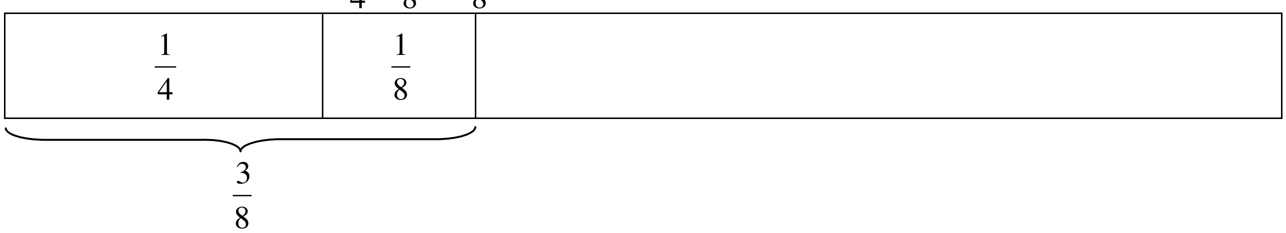
Alijah can clean the kids room in 4 minutes. Tarek can clean the room in 8 minutes. If they start to clean the room together for 2 minutes, how long will it take Tarek to finish by himself?

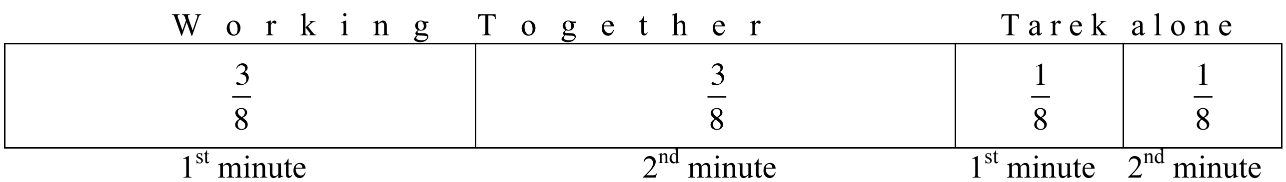
Alijah can clean $\frac{1}{4}$ of the room in one minute.

Tarek can clean $\frac{1}{8}$ of the room in one minute.



Together they can set up $\frac{1}{4}+\frac{1}{8}$ or $\frac{3}{8}$ of the room in one minute.





Therefore, it will take Tarek 2 minutes working alone to finish cleaning the kids room.

When we set up the equation, we need to define our variable.

Let x = the amount of time it takes Tarek to finish cleaning the kids room

$$\left(\frac{1}{4}+\frac{1}{8}\right)2+\left(\frac{1}{8}\right)x=1$$
$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}x=1$$
$$\frac{3}{4}-\frac{3}{4}+\frac{1}{8}x=1-\frac{3}{4}$$
$$(8)\frac{1}{8}x=\frac{1}{4}(8)$$
$$\boxed{x=2}$$

$$\left(\frac{1}{4}+\frac{1}{8}\right)2+\left(\frac{1}{8}\right)x=1$$
$$\left(\frac{1}{4}\right)2+\left(\frac{1}{8}\right)2+\left(\frac{1}{8}\right)x=1$$
$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}x=1$$
$$8\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}x\right)=8(1)$$
$$4+2+x=8$$
$$6+x=8$$
$$6+x-6=8-6$$
$$\boxed{x=2}$$

$\therefore$ It will take Tarek 2 minutes to finish cleaning the kids room.